Chapter – 1 Relations and Functions

Exercise 1.1

1. Show that the relation R in the set {1,2,3} given by R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)} is reflexive but neither symmetric nor transitive. **[Example 4]**
2. Show that the relation R in the set R of real numbers, define as R = {(a, b) : a $\leq b^2$} is neither reflexive nor symmetric nor transitive. **[Q 2]**
3. Show that the relation R in R defined as R = {(a, b) : a $\leq b$}, is reflexive and transitive but not symmetric. **[Q 4]**
4. Check whether the relation R in R defined by R = {(a, b) : a $\leq b^3$} is reflexive, symmetric or transitive. **[Q 5]**
5. Show that the R in the set {1, 2, 3} given by R = {(1, 2), (2, 1)} symmetric but neither reflexive nor transitive. **[Q 6]**
6. Show that the relation R in the set A = {1,2,3,4,5} given by R = {(a, b) : |a – b| is even}, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5} is related to any element of {2, 4}. **[Q 8]**
7. Show that each of the relation R in the set A = {x $\in Z$: $0 \leq x \leq 12$}. given by
   (a) R = {(a, b) : |a – b| is a multiple of 4}
   (b) R = {(a, b) : a = b}
   Is an equivalence relation. Find the set of all elements related to 1 in each case. **[Q 9]**
8. Show that the relation R in the set A of points in a plane given by R = {(P, Q) : distance of the point P from the origin is same as the distance of the point Q from the origin}, is an equivalence relation. Further, show that the set of all points related to a point P $\neq (0,0)$ is the circle passing through P with origin as centre. **[Q 11]**
9. Show that the relation R defined in the set A of all triangle as R = {(T_1, T_2) :T_1 is similar to T_2}, is equivalence relation. Consider three right angle trianglesT_1 with sides 3, 4, 5, T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1, T_2 and T_3 are related? **[Q 12]**

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10. Show that the relation R defined in the set A of all polygons as R = {(P_1, P_2) : P_1 and P_2 have same number of sides}, is an equivalence relation. What is the set of all elements in A related to the right angle T with sides 3, 4 and 5? **[Q 13]**

Exercise 1.2

11. Show that the function f: R →R, defined as f(x) = x^2, is neither one-one nor onto. **[Example 11]**

12. Show that f: N →N, given by f(x) = $\begin{cases} x+1, if\ x\ is\ odd \\ x-1, if\ x\ is\ even \end{cases}$ is both one-one and onto. **[Example 12]**

13. Prove that the Greatest Integer Function f:R →R given by f(x) = [x], is neither one-one nor onto, where [x] denotes the greatest integer less than or equal to x. **[Q 3]**

14. Let A = {1, 2, 3}, B = {4, 5, 6} and let f = {(1, 4), (2, 5), (3, 6)} be function from A to B. Show that f is one-one. **[Q 6]**

15. In each of the following cases, state whether the function is one-one, onto or bijective. Justify your answer.

(a) f: R →R defined by f(x) = 3 – 4x.

(b) f: R →R defined by f(x) = 1 + x^2. **[Q 7]**

16. Let A and B be sets. Show that f : A x B →B x A such that f(a, b) = (b, a) is bijective function. **[Q 8]**

17. Let f : N →N be defined by f(n) = $\begin{cases} \frac{n+1}{2}, if\ n\ is\ odd \\ \frac{n}{2}, if\ n\ is\ even \end{cases}$ for all n = N.

State whether the function f is bijective. Justify your answer. **[Q 9]**

18. Let A = R – {3} and B = R – {1}.consider the function f : A → B defined by f(x) = $\left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer. **[Q 10]**

Related to Miscellaneous Exercise

19. Let A = {-1, 0, 1, 2, b = {-4, -2, 0, 2} and f, g:A→ B be functions defined by f(x) = $x^2 - x$, x ∈ A and $g(x) = 2\left|x - \frac{1}{2}\right| - 1, x \in A$. are f and g equal? Justify your answer. (hind one may note that two functions f: A → B and g: A → B such that f(a) = g(a) ∀ $a \in A$, are called equal functions). **[Q 5]**

20. Let A = {1,2,3}. The number of relations containing (1, 2) and (1, 3) which are reflexive and symmetric but not transitive is – **[Q 6]**

(a) 1

(b) 2

(c) 3

(d) 4

21. Let A = {1, 2, 3}. Then number of equivalence relations containing (1, 2) is

(a) 1

(b) 2

(c) 3

(d) 4

[Q 7]

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Answers

Exercise 1.1

1. R is reflexive, not transitive

4. Neither reflexive nor symmetric nor transitive.

7. (i){1,5,9} (ii){1}

9. T_1 is related T_3.

10. The set of all triangles.

Exercise 1.2

15. (a) One-one and onto (b) Neither one-one nor onto

17. no

18. Yes

Miscellaneous Exercise

19. Yes

20. A

21. B

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Exercise 2.1

1. Find the principal values of the following: $\cos^{-1}\left(-\frac{1}{2}\right)$ **[Q 5]**

2. Find the principal values of the following: $\tan^{-1}(-1)$ **[Q 6]**

3. Find the value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$. **[Q 11]**

Exercise 2.2

4. Express $\tan^{-1}\frac{\cos x}{1-\sin x}, \frac{-3\pi}{2} < x < \frac{\pi}{2}$ in the simplest form. **[Example 4]**

5. Write the in the simplest form: $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right), x \neq 0$ **[Q 3]**

6. Write the in the simplest form: $\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right), x < x < \pi$ **[Q 4]**

7. Write the in the simplest form: $\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right), x < x < \pi$ **[Q 5]**

8. Write the in the simplest form: $\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right), |x| < a$ **[Q 6]**

9. Find the values of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$ **[Q 8]**

10. Find the values of

$\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right], |x| < 1, y > 0$ and $xy < 1$ **[Q 9]**

11. Find the values of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ **[Q 10]**

12. $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ **[Q 11]**

13. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ is equal to **[Q 13]**

(i) $\frac{7\pi}{6}$

(ii) $\frac{5\pi}{6}$

(iii) $\frac{\pi}{3}$

(iv) $\frac{\pi}{6}$

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Miscellaneous Exercise

14. Simplify $\tan^{-1}\left[\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right], if\ \frac{a}{b}\tan x > -1.$ **(Old NCERT)**

15. Find the value of the following: $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$ **[Q 2]**

16. Prove that: $\cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{56}{65}$ **[Q 6]**

17. Prove that: $\tan^{-1}\frac{63}{16} = \sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5}$ **[Q 7]**

18. Prove that: $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x \in [0,1]$ **[Q 8]**

19. Prove that: $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \frac{x}{2}, \in \left(0, \frac{\pi}{4}\right)$ **[Q 9]**

20. Prove that: $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x, -\frac{1}{\sqrt{2}} \leq x \leq 1$ **[Q 10]**

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Answers

Exercise 2.1

1. Ans. $\frac{2\pi}{3}$

2. Ans. $-\frac{\pi}{4}$

3. Ans. $\frac{3\pi}{4}$

Exercise 2.2

4. Ans. $\frac{\pi}{4}+\frac{x}{2}$

5. Ans. $\frac{1}{2}\tan^{-1} x$

6. Ans. $\frac{x}{2}$

7. Ans. $\frac{\pi}{4}-x$

8. Ans. $\sin^{-1}\frac{x}{a}$

9. Ans. $\frac{\pi}{4}$

10. Ans. $\frac{x+y}{1-xy}$

11. Ans. $\frac{\pi}{3}$

12. Ans. $\frac{-\pi}{4}$

13. (ii)

Miscellaneous Exercise

14. Ans. $\tan^{-1}\frac{a}{b}-x$

15. Ans. $\frac{\pi}{6}$

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Chapter-3 MATRIX

Exercise 3.1

1. Construct a 3 x 2 matrix whose elements are given by a_{ij} = ½ |i – 3j|. **[Example 3]**

2. If $\begin{bmatrix} x+3 & z+4 & 2y-7 \\ -6 & a-1 & 0 \\ b-3 & -21 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 6 & 3y-2 \\ -6 & -3 & 2c+2 \\ 2b+4 & -21 & 0 \end{bmatrix}$. Find the values of a, b, c, x, y and z. **[Example 4]**

3. The number of all possible matrices of order 3 x 3 with each entry 0 or 1 is

(a) 27

(b) 18

(c) 81

(d) 512 **[Q 10]**

Exercise 3.2

4. Simplify $\cos\theta \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} sin\theta & -cos\theta \\ cos\theta & sin\theta \end{bmatrix}$ **[Q 6]**

5. Find X, If Y = $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and 2X + Y = $\begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$. **[Q 8]**

6. Given $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$, find the values of x, y, z and w. **[Q 12]**

7. If A = $\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ $and\ I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that A^2 = kA – 2I **[Q 17]**

8. If A = $\begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that I + A = (I – A) $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ **[Q 18]**

9. A trust fund has Rs 30,000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide Rs 30,000 among the two types of bonds. If the trust fund must obtain an annual total interest of :

(a) Rs 1800

(b) Rs 2000 **[Q 19]**

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Assume X,Y,Z, W and P are matrices of order 2 x n , 3 x k , 2 x p , n x 3 and p x k respectively .Choose the correct answer

10. The restriction on n, k and p so that PY + WY will be defined are:

(a) k =3, p = n

(b) k is arbitrary, p = 2

(c) p is arbitrary, k = 3

(d) k = 2, p = 3 **[Q 21]**

11. if n = p then the order of the matrix 7X – 5Z is:

(a) p x 2

(b) 2 x n

(c) n x 3

(d) p x n **[Q 22]**

Exercise 3.3

12. Express the matrix B = $\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix. **[Example 22]**

13. If A, B are symetirc matrices of same order, then AB – BA is a

(a) Skew symmetric matrix

(b) Symmetric matrix

(c) Zero matrix

(d) Identity matrix **[Q 11]**

14. If A = $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, the A + A' = 1, if the value of α is

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{3}$

(c) π

(d) $\frac{3\pi}{2}$ **[Q 12]**

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Miscellaneous Exercise

15. If A and B are symmetric matrices prove that AB – BA is a skew symmetric matrix. **[Q 1]**

16. Show that the matrix B'AB is symmetric or skew symmetric according as A is symmetric or skew symmetric. **[Q 2]**

17. Find the values of x, y, z if the matrix A = $\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfy the equation A'A = I. **[Q 3]**

18. A manufacturer produces three products x, y, z which he sells in two markets. Annual sales are indicated below:

Market		Products	
I	10,000	2,000	18,000
II	6,000	20,000	8,000

(a) If unit sale prices x, y and z are Rs 2.50, Rs 1.50 and Rs 1.00, respectively, find the total revenue in each market with the help of matrix algebra.

(b) If the unit costs of the above three commodities are Rs 2.00, Rs 1.00 and 50 paise respectively. Find the gross profit. **[Q 7]**

19. Find the matrix X so that X $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$. **[Q 8]**

20. If A is square matrix such that A^2 = A ,Then $(I + A)^3$ – 7 A is equal to

(a) A

(b) I – A

(c) I

(d) 3A **[Q 11]**

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Answers

Exercise 3.1

1. $\begin{bmatrix} 1 & \frac{5}{2} \\ \frac{1}{2} & 2 \\ 0 & \frac{3}{2} \end{bmatrix}$

2. $a = -2, b = -7, c = -1, x = -3, y = -5, z = 2$

3. (d)

Exercise 3.2

4. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

5. $X = \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$

6. $x = 2, y = 4, w = 3, z = 1$

7. $k = 1$

8.

9. (a) $Rs\ 15000, Rs\ 15000$ (b) $Rs\ 5000, Rs\ 25000$

10. (a)

11. (b)

Exercise 3.3

13. A **14.** B

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Miscellaneous Exercise

17. $x = \pm\frac{1}{\sqrt{2}}, y = \pm\frac{1}{\sqrt{6}}, z = \pm\frac{1}{\sqrt{3}}$

18. (a) total revenue in the market – I = Rs 46000 ; Total revenue in the market – II = Rs 53000 (b) Rs 15000, Rs 17000

19. $X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$

20. (c)

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Chapter-4 DETERMINANTS

Exercise 4.1

1. Find the values of x for which $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$. **[Example 5]**

2. If A = $\begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then show that |2A| = 4|A| **[Q 3]**

3. If A = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, then show that |3A| = 27|A|. **[Q 4]**

4. If A = $\begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$, find |A| **[Q 6]**

Exercise 4.2

5. Find area of the triangle with vertices at the point given in each of the following: (2, 7), (1,1),(10, 8) **[Q 1(ii)]**

6. Find values of k if area of triangle is 4sq. units and vertices are (k, 0), (4, 0), (0, 2) **[Q 3(i)]**

7. Find equation of line joining (1, 2) and (3, 6) using determinants. **[Q 4(i)]**

8. If area of triangle is 35 sq units with vertices (2, -6), (5, 4) and (k, 4). Then k is

(a) 12

(b) -2

(c) 12, -2

(d) 12 , -2 **[Q 5]**

Exercise 4.3

9. Write the minors and cofactors of the element of determinants :

$\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$ **[Q 2(ii)]**

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Exercise 4.4

10. Find the inverse of the matrix: $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ **[Q 10]**

11. For the matrix A = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$ show that $A^3 - 6A^2 + 9A - 41 = 0$ and hence find A^{-1}. **[Q 15]**

12. Let A be a nonsingular square matrix of order 3 x 3. Ten |adj A| is equal to

(a) |A|

(b) $|A|^2$

(c) $|A|^3$

(d) 3|A| **[Q 17]**

13. If A is an invertible matrix of order 2, then det (A^{-1}) is equal to

(a) det(A)

(b) $\frac{1}{\det(A)}$

(c) 1

(d) 0 **[Q 18]**

Exercise 4.5

14. Solve the following system of equations by matrix method.

3x – 2y + 3z = 8

2x + y – z = 1

4x – 3y + 2z = 4 **[Example 17]**

15. The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method. **[Example 18]**

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Miscellaneous Exercise

16. Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations

$x - y + 2z = 1$

$2y - 3z = 1$

$3x - 2y + 4z = 2$ **[Example 19]**

17. Solve the system of the following equations

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$

$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$$

$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$ **[Q 7]**

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Answers

Exercise 4.1

1. $\pm 2\sqrt{2}$

4. 0

Exercise 4.2

5. 47/2

6. 0, 8

7. y = 2x

8. (d)

Exercise 4.3

9. $M_{11} = 11, M_{12} = 6, M_{13} = 3, M_{21} = 4, M_{22} = 2, M_{23} = 1, M_{31} = -20, M_{32} = -13, M_{33} = 5$

$A_{11} = 11, A_{12} = -6, A_{13} = 3, A_{21} = 4, A_{22} = 2, A_{23} = -1, A_{31} = -20, A_{32} = 13, A_{33} = 5.$

Exercise 4.4

10. $\begin{matrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{matrix}$

11. $A^{-1} = \frac{1}{11}\begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$

12. (b)

13. (b)

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Exercise 4.5

14. x= 1, y = 2 and z = 3

15. x= 1, y = 2 and z = 3

Miscellaneous Exercise

16. x= 0, y = 5 and z = 3

17. x= 2, y = 3 and z = 5

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Chapter-5 CONTINUITY AND DIFFERENTIABILITY

Exercise 5.1

1. Find all points of discontinuity of *f*, where *f* is defined by

$$f(x) = \begin{cases} |x| + 3, if\ x \leq -3 \\ -2x, if\ -3 < x < 3 \\ 6x + 2, if\ x \geq 3 \end{cases} \quad \textbf{[Q 7]}$$

2. Find all points of discontinuity of *f*, where *f* is defined by

$$f(x) = \begin{cases} \frac{|x|}{x}\ if\ x \neq 0 \\ 0, if\ x = 0 \end{cases} \quad \textbf{[Q 8]}$$

3. find the relationship between a and b so that the function f defined by

$$f(x) = \begin{cases} ax + 1,\ if\ x \leq 3 \\ bx + 3, if\ x > 3 \end{cases}$$ Continuous at x = 3 **[Q 17]**

4. Show that the function defined by g(x) = x – [x] is discontinuous at all integrate points. Hence [x] denotes the greatest integer less than or equal to x. **[Q 19]**

5. Is the function defined by $f(x) = x^2 - \sin x + 5$ continuous at $x = \pi$? **[Q 20]**

6. Find the value of k so that the function f is continuous at the indicated point:

$$f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x}, if\ x \neq \frac{\pi}{2} \\ 3, \quad if\ x = \frac{\pi}{2} \end{cases} at\ x = \frac{\pi}{2} \quad \textbf{[Q 26]}$$

7. Find the value of k so that the function f is continuous at the indicated point:

$$f(x) = \begin{cases} kx^2,\ if\ x \leq 2 \\ 3, \quad if\ x > 2 \end{cases} at\ x = 2 \quad \textbf{[Q 27]}$$

8. Find the value of k so that the function f is continuous at the indicated point:

$$f(x) = \begin{cases} kx + 1,\ if\ x \leq \pi \\ \cos x,\ if\ x = \pi \end{cases} at\ x = \pi \quad \textbf{[Q 28]}$$

9. Find the value of k so that the function f is continuous at the indicated point:

$$f(x) = \begin{cases} kx + 1,\ if\ x \leq 5 \\ 3x - 5,\ if\ x > 5 \end{cases} at\ x = 5 \quad \textbf{[Q 29]}$$

Exercise 5.2

10. Prove that the function f given by f(x) = |x – 1|, x∈ R is not differentiable at x = 1. **[Q 9]**

Exercise 5.3

11. Find the dy/dx $x^2 + xy + y^2 = 100$ **[Q 5]**

12. Find the dy/dx $\sin^2 y + \cos xy = k$ **[Q 7]**

13. Find the dy/dx $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ **[Q 9]**

14. Find the dy/dx $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$ **[Q 11]**

15. Find the dy/dx $y = \sec^{-1}\left(\frac{1}{2x^2-1}\right), 0 < x < \frac{1}{\sqrt{2}}$ **[Q 15]**

Exercise 5.4

16. Differentiate the following w.r.t. x. e^{x^3} **[Q 3]**

17. Differentiate the following w.r.t. x. $\sin(tan^{-1} e^{-x})$ **[Q 4]**

18. Differentiate the following w.r.t. x. $\sqrt{e^{\sqrt{x}}}, x > 0$ **[Q 7]**

Exercise 5.5

19. Differentiate the following w.r.t. x. $\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$ **[Q 2]**

20. Differentiate the following w.r.t. x. $x^x - 2^{\sin x}$ **[Q 4]**

21. Differentiate the following w.r.t. x. $\left(x+\frac{1}{x}\right)^x + x^{\left(x+\frac{1}{x}\right)}$ **[Q 6]**

22. Differentiate the following w.r.t. x. $(\log x)^x + x^{\log x}$ **[Q 7]**

23. Differentiate the following w.r.t. x. $(\sin x)^x + \sin^{-1}\sqrt{x}$ **[Q 8]**

24. Differentiate the following w.r.t. x. $(x \cos x)^x + (x \sin x)^{1/x}$ **[Q 11]**

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Exercise 5.6

25. Find dy/dx: x = a$(\theta - \sin\theta), y = a(1 + \cos\theta)$ **[Q 6]**

26. Find dy/dx: x = $\frac{sin^3 t}{\sqrt{\cos 2t}}, y = \frac{cos^3 t}{\sqrt{\cos 2t}}$ **[Q 7]**

27. Find dy/dx: x = a(cot t + log tan t/2) , y = a sin t **[Q 8]**

28. If x = $\sqrt{a^{\sin^{-1} t}}, y = \sqrt{a^{\cos^{-1} t}}$, show that dy/dx = $-\frac{y}{x}$ **[Q 11]**

Exercise 5.7

29. If y = $\sin^{-1}$x, show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$ **[Example 38]**

30. If y = 3 cos (log x) + 4 sin (log x), show that $x^2y_2 + xy_1 + y = 0$ **[Q 13]**

31. If y = $(\tan^{-1}x)^2$, show that $(x^2 + 1)^2 y_2 + 2x(x^2 + 1) y_1 = 2$. **[Q 17]**

Miscellaneous Exercise

32. Differentiate the following w.r.t. x.

(a) $\tan^{-1}(\frac{\sin x}{1+\cos x})$ **[Example 40(i)]**

(b) $\sin^{-1}(\frac{2^{x+1}}{1+4^x})$ **[Example 40(iii)]**

33. Differentiate $\sin^2$ x w.r.t. $e^{\cos x}$. **[Example 43]**

34. Differentiate w.r.t.x. $\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right], 0 < x < \frac{\pi}{2}$ **[Q 6]**

35. Differentiate w.r.t.x $(\sin x - \cos x)^{(\sin x - \cos x)}, \frac{\pi}{4} < x < \frac{3\pi}{4}$ **[Q 9]**

36. Find dy/dx, if y = 12 (1 – cos t), x = 10(t – sin t), $-\frac{\pi}{2} < t < \frac{\pi}{2}$ **[Q 12]**

37. If x $\sqrt{1+y} + y\sqrt{1+x} = 0, for\ , -1 < x < 1$, prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$.

[Q 14]

38. If cos y = x cos(a+y), with cos a $\neq \pm 1$, prove that dy/dx = $\frac{cos^2(a+y)}{\sin a}$ **[Q 16]**

39. If x = a (cot t + t sin t) and y = a(sin t – t cot t), find d^2y/dx^2. **[Q 17]**

40. if y = $e^{a\cos^{-1}x}, -1 \leq x \leq 1, show\ that$ $(1 - x^2)\ d^2y/dx^2 - x\ dy/dx - a^2y = 0$.

[Q 22]

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Answers

Exercise 5.1

1. discontinuous at x = 3

2. discontinuous at x = 0

3. a = b +2/3

5. f is continuous at x = π

6. k = 6

7. k = ¾

8. k = -2/ π

9. k = 9/5

Exercise 5.3

11. $-\frac{(2x+y)}{(x+2y)}$

12. $\frac{y \sin xy}{\sin 2y - x \sin xy}$

13. $\frac{2}{1+x^2}$

14. $\frac{2}{1+x^2}$

15. $-\frac{2}{\sqrt{1-x^2}}$

Exercise 5.4

16. $3x^2e^{x^3}$

17. $-\frac{e^{-x}\cos(\tan^{-1}e^{-x})}{1+e^{-2x}}$

18. $\frac{e^{\sqrt{x}}}{4\sqrt{xe^{\sqrt{x}}}}, x > 0$

Exercise 5.5

19. $\frac{1}{2}\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}\left[\frac{1}{x-1}+\frac{1}{x-2}-\frac{1}{x-3}-\frac{1}{x-4}-\frac{1}{x-5}\right]$

20. $x^x(1+\log x) - 2^{\sin x}\cos x \log 2$

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21. $\left(x+\frac{1}{x}\right)^x\left[\frac{x^2-1}{x^2+1}\right]+\ log\ \left(x+\frac{1}{x}\right)+\ x^{1+\frac{1}{x}}\left(\frac{x+1-log\ x}{x^2}\right)$

22. $(\log x)^{x-1}\ [1 + \log x.\ \log (\log x)] + 2x^{\log x-1}\ .\ \log x$

23. $(\sin x)^x\ (\ x \cot x + \log \sin x) + ½\ \frac{1}{\sqrt{1-x^2}}$

24. $(x \cos x)^x\ [1 – x \tan x + \log (x \cos x)\ ] + (x \sin x)^{1/x}\left[\frac{x cot\ x+1-log\ (x\ sin\ x)}{x^2}\right]$

Exercise 5.6

25. $-\cot\frac{\theta}{2}$

26. $-\cot 3t$

27. $\tan t$

Miscellaneous Exercise

32. (a) ½ (b) $\frac{2^{x+1}\log 2}{1+4^x}$

33. $-\frac{2\cos x}{e^{\cos x}}$

34. $\frac{1}{2}$

35. $(\sin x – \cos x)^{\sin x – \cos x}\ (\cos x + \sin x).(1 + \log (\sin x – \cos x))$, $\sin x > \cos x$

36. $\frac{6}{5}\cot\frac{t}{2}$

39. $\frac{sec^3 t}{at}, 0 < t < \frac{\pi}{2}$

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Chapter-6 APPLICATION OF DERIVATIVES

Exercise 6.1

1. Find the volume of a cube is increasing at a rate of 9 cubic centimeters per second. How fast is the surface area increasing when the length of an edge is 10 centimeters? **[Example 2]**
2. The length x of a rectangle is decreasing at the rate of 3 cm/minute and the width y is increasing at the rate of 2cm/minute. When x = 10 cm and y = 6cm, find the rates of change of (a) the perimeter and (b) the area of the rectangle. **[Example 4]**
3. The total cost C(x) in Rupees, associated with the production of x units of an item is given by $C(x) = 0.005\ x^3 - 0.02\ x^2 + 30x + 5000$. Find the marginal cost when 3 units are produced, where by marginal cost we mean the instantaneous rate of change of cost at any level of output. **[Example 5]**
4. The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$. Find the marginal revenue, when x = 5, where by marginal revenue we mean the rate of change of total revenue with respect to the number of items sold at an instant. **[Example 6]**
5. The radius of a circle is increasing uniformly at the rate of 3 cm/s. find the rate at which the area of the circle is increasing when the radius is 10 cm. **[Q 3]**
6. A stone is dropped into a quiet lake and waves move in circles at the speed of 5 cm/s. at the instant when the radius of the circular wave is 8 cm, how fast is the enclosed area increasing? **[Q 5]**
7. A balloon, which always remains spherical on inflation, is being inflated by pumping in 900 cubic centimeter of gas per second. Find the rate at which the radius of the balloon increases when the radius is 15 cm. **[Q 8]**
8. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 cm/s. how fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall? **[Q 10]**
9. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate. **[Q 11]**

10. The radius of an air bubble is increasing at the rate of ½ cm/s. At what rate is the volume of the bubble increasing when the radius is 1 cm? **[Q 12]**

11. A balloon, which always remains spherical, has a variable diameter 3/2 (2x +1). Find the rate of change of its volume with respect to x. **[Q 13]**

12. Sand is pouring form a pipe at the rate of 12 cm^3/s. the falling sand forms a cone n the pouring in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm? **[Q 14]**

Exercise 6.2

13. Find the intervals in which the function f given by $f(x) = 4x^3 - 6x^2 - 72x + 30$ is (a) increasing (b) decreasing. **[Example 11]**

14. Find intervals in which the function given by $f(x) = \sin 3x$, $x \in \left[0, \frac{\pi}{2}\right]$ is (a) increasing (b) decreasing **[Example 12]**

15. Find the intervals in which the function *f* given by $f(x) = \sin x + \cos x$, $0 \leq x \leq 2\pi$ is increasing or decreasing. **[Example 13]**

16. Find the intervals in which the following functions are strictly increasing or decreasing: $(x + 1)^3 (x - 3)^3$ **[Q 6(e)]**

17. Show that $y = \log(1+x) - \frac{2x}{2+x}$, $x > -1$, is an increasing function of x throughout its domain. **[Q 7]**

18. Find the values of x for which $y = [x(x - 2)]^2$ is an increasing function. **[Q 8]**

19. Prove that $y = \frac{4 \sin \theta}{(2 \cos \theta)} - \theta$ *is an increasing fucniton of* θ *in* $\left[0, \frac{\pi}{2}\right]$. **[Q 9]**

20. Prove that the function f given by $f(x) = x^2 - x + 1$ is neither strictly increasing nor strictly decreasing on (-1, 1). **[Q 11]**

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Exercise 6.3

21. If length of three sides of a trapezium other than base are equal to 10cm, then find the area of the trapezium when it is maximum. **[Example 25]**

22. A square piece of tin of side 18 cm is to be made into a box without top, by cutting a square from each corner and folding up the flaps to form the box. What should be the side of the square to be cut off so that the volume of the box is the maximum possible. **[Q 17]**

23. A rectangular sheet of tin 45 cm by 24 cm is to be made into a box without top, by cutting off square for each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is maximum? **[Q 18]**

24. Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area. **[Q 19]**

25. Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base. **[Q 20]**

26. Of all the closed cylindrical cans (right circular), of a given volume of 100 cubic centimeters, find the dimensions of the can which has the minimum surface area? **[Q 21]**

27. A wire of length 28 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle, what should be the length of the two pieces so that combined area of the square and the circle is minimum? **[Q 22]**

28. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is 8/27 of the volume of the sphere. **[Q 23]**

29. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ time the radius of the base. **[Q 24]**

30. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$. **[Q 25]**

31. Show that semi-vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$. **[Q 26]**

Miscellaneous Exercise

32. Manufacturer can sell x items at a price of rupees $\left(5 - \frac{x}{100}\right)$each. The cost price of x items is Rs $\left(\frac{x}{5} + 500\right)$. Find the number of items he should sell to earn maximum profit. **[Example 37]**

33. The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm per second. How fast is the area decreasing when the two equal sides are equal to the base. **[Q 2]**

34. Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. With its vertex at one end of the major axis. **[Q 5]**

35. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m^3. If building of tank costs Rs 70 per sq meters for the base and Rs 45 per square meters for sides. What is the cost of least expensive tank? **[Q 6]**

36. The sum of the perimeter of a circle and square is k, where k is some constant. prove that the sum of their areas is least when the side of square is double the radius of the circle. **[Q 7]**

37. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. find the dimensions of the window to admit maximum light through the whole opening. **[Q 8]**

38. A point on the hypotenuse of a triangle is at distance a and b from the sides of the triangle. Show that the minimum length of the hypotenuse is $(a^{2/3} + b^{2/3})^{3/2}$**[Q 9]**

39. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is 4r/3. **[Q 12]**

40. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also, find the maximum volume. **[Q 14]**

41. Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi vertical angle α is one-third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi h^3 tan^2\alpha$. **[Q 15]**

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Answers

Exercise 6.1

1. $3.6 cm^2/s$

2. (a) -2 cm/min, (b) $2\ cm^2/min$

3. 30.02

4. Rs. 66

5. $60\pi\ cm^2/s$

6. $80\pi\ cm^2/s$

7. $\frac{1}{\pi}$ cm/s

8. 8/3 cm/s

9. (4, 11) and $(-4, \frac{-33}{3})$

10. $2\pi\ cm^3/s$

11. $\frac{22}{8}\pi\ (2x+1)^2$

12. $\frac{1}{48\pi}$ cm/s

Exercise 6.2

13. is strictly increasing in the intervals $(-\infty, -2)\ and\ (3, \infty)$, strictly decreasing in the interval (-2,3)

14. f is increasing on $[0, \frac{\pi}{6}]$ and decreasing on $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$

15. f is strictly increasing in the intervals $\left[0, \frac{\pi}{4}\right] and\ \left(\frac{5\pi}{4}, 2\pi\right]$, f is strictly decreasing in $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$.

16. Ans. strictly increasing in (1, 3) and (3, ∞), strictly decreasing in $(-\infty, -1)\ and\ (-1, 1)$

17.

18. 0<x<1 and x> 2

Exercise 6.3

21. $75\sqrt{3}\ cm^2$

22. 3 cm

23. x= 5 cm

24.

25.

26. radius = $\left(\frac{50}{\pi}\right)^{\frac{1}{3}}\ cm\ and\ height = 2\left(\frac{50}{\pi}\right)^{\frac{1}{3}}$ cm

27. $Ans. \frac{112}{\pi+4} cm, \frac{28\pi}{\pi+4} cm$

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Miscellaneous Exercise

32. $x = 240$

33. $b\sqrt{3}$ cm^2/s

34. $\frac{3\sqrt{3}}{4}ab$

35. $Rs.1000$

36.

37. $length = \frac{20}{\pi+4}m, breadth = \frac{10}{\pi+4}m$

38.

39.

40. $\frac{4\pi R^3}{3\sqrt{3}}$

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Chapter-7 Integrals

Exercise-7.1

1. $\int \frac{x^3+3x+4}{\sqrt{x}} dx$ **[Q 12]**

2. $\int \frac{2-3\sin x}{cos^2 x} dx$ **[Q 20]**

Exercise-7.2

3. $\int \frac{\tan^4 \sqrt{x} sec^2 \sqrt{x}}{\sqrt{x}} dx$ **[Example 5(iii)]**

4. $\int \frac{\sin x}{\sin(x+a)} dx$ **[Example 6(ii)]**

5. $\int \frac{1}{1+\tan x} dx$ **[Example 6(iii)]**

6. $\int \frac{e^{2x}-1}{e^{2x}+1} dx$ **[Q 19]**

7. $\int \frac{\cos x}{\sqrt{1+\sin x}} dx$ **[Q 28]**

8. $\int \frac{1}{1+\cot x} dx$ **[Q 32]**

9. $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$ **[Q 34]**

Exercise-7.3

10. $\int \text{Sin}^2(2x + 5)\, dx$ **[Q 1]**

11. $\int \sin^3 (2x + 1)\, dx$ **[Q 4]**

12. $\int \sin 4x \sin 8x\, dx$ **[Q 7]**

13. $\int \frac{\cos 2x-\cos 2\alpha}{\cos x-\cos \alpha} dx$ **[Q 13]**

14. $\int \frac{\cos x-\sin x}{1+\sin 2x} dx$ **[Q 14]**

15. $\int \tan^3 2x \sec 2x\, dx$ **[Q 15]**

16. $\int \frac{1}{\sin x cos^3 x} dx$ **[Q 19]**

17. $\int \frac{\cos 2x}{(\cos x+\sin x)^2} dx$ **[Q 20]**

18. $\int \frac{1}{\cos(x-a)\cos(x-b)} dx$ **[Q 22]**

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Exercise-7.4

19. $\int \frac{dx}{3x^2+13x-10}\,dx$ **[Example 9(ii)]**

20. $\int \frac{x+3}{\sqrt{5-4x-x^2}}\,dx$ **[Example 10(ii)]**

21. $\int \frac{1}{9x^2+6x+5}\,dx$ **[Q 11]**

22. $\int \frac{x+3}{x^2-2x-5}\,dx$ **[Q 22]**

Exercise-7.5

23. Find $\int \frac{x^2+1}{x^2-5x+6}\,dx$ **[Example 12]**

24. Find $\int \frac{3x-2}{(x+1)^2(x+3)}\,dx$ **[Example 13]**

25. Find $\int \frac{(3\sin\phi-2)\cos\phi}{5-cox^2\phi-4sin\phi}\,d\phi$ **[Example 15]**

26. Find $\int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx$ **[Example 16]**

27. $\int \frac{3x-1}{(x-1)(x-2)(x-3)}\,dx$ **[Q 3]**

28. $\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}\,dx$ **[Q 18]**

29. $\int \frac{2x}{(x^2+1)(x^2+3)}\,dx$ **[Q 19]**

30. $\int \frac{1}{x(x^4-1)}\,dx$ **[Q 20]**

31. $\int \frac{1}{(e^x-1)}\,dx$ [hint: Put $e^x = t$] **[Q 21]**

Exercise-7.6

32. $\int \frac{x\cos^{-1}x}{\sqrt{1-x^2}}\,dx$ **[Q 11]**

33. $\int \frac{xe^x}{(1+x)^2}\,dx$ **[Q 17]**

34. $\int e^x\left(\frac{1+\sin x}{1+\cos x}\right)\,dx$ **[Q 18]**

35. $\int \frac{(x-3)e^x}{(x-1)^3}\,dx$ **[Q 20]**

36. $\int e^{2x}\sin x\,dx$ **[Q 21]**

37. $\int \sin^{-1}\left(\frac{2x}{1+x^2}\right)\,dx$ **[Q 22]**

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Exercise-7.7

38. Integrate the function: $\sqrt{x^2 + 4x - 5}$. **[Q 6]**

Exercise-7.10

39. Evaluate $\int_{-1}^{2} |x^3 - x| dx$ **[Example 28]**

40. Evaluate $\int_0^{\pi} \frac{x \sin x}{1 + cos^2 x} dx$ **[Example 30]**

41. Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}}$ **[Example 33]**

42. Evaluate $\int_0^{\pi/2} \log \sin x \, dx$ **[Example 34]**

By using the propertied of definite integrals, evaluate the integrals:

43. $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ **[Q 2]**

44. $\int_{-5}^{5} |x + 2| \, dx$ **[Q 5]**

45. $\int_0^{\frac{\pi}{4}} \log(1 + \tan x) \, dx$ **[Q 8]**

46. $\int_0^{\frac{\pi}{2}} (2 \log \sin x - \log \sin 2x) \, dx$ **[Q 10]**

47. $\int_0^{\pi} \log(1 + \cos x) \, dx$ **[Q 16]**

Miscellaneous Exercise

48. Find $\int \frac{x^4 dx}{(x-1)(x^2+1)}$ **[Example 37]**

49. Find $\int \left[\log(\log x) + \frac{1}{(\log x)^2}\right] dx$ **[Example 38]**

50. Find $\int \left[\sqrt{\cot x} + \sqrt{\tan x}\right] dx$ **[Example 39]**

51. Evaluate $\int_0^{\pi} \frac{x dx}{a^2 cos^2 x + b^2 sin^2 x}$ **[Example 42]**

52. $\int \frac{\sin x}{\sin(x-a)} dx$ **[Q 7]**

53. $\int \frac{1}{\cos(x+a) \cos(x+b)} \, dx$ **[Q 11]**

54. $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} \, dx$ **[Q 19]**

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55. $\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}\,dx$ **[Q 22]**

56. $\int \frac{\sqrt{x^2+1}[\log(x^2+1)-2\log x]}{x^4}dx$ **[Q 23]**

57. $\int_0^{\frac{\pi}{4}} \frac{\sin x+\cos x}{9+16\sin 2x}dx$ **[Q 29]**

58. $\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x)\,dx$ **[Q 30]**

59. $\int_0^{\pi} \frac{x\tan x}{\sec x+\tan x}dx$ **[Old NCERT]**

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Answers

Exercise-7.1

1. $\frac{2}{7}x^{\frac{7}{2}} + 2x^{\frac{3}{2}} + 8\sqrt{x} + C$

2. $2\tan x - 3\sec x + C$

Exercise-7.2

3. $\frac{2}{5}tan^5\sqrt{x} + C$

4. $x\cos a + a\cos a - (\sin a)log\,|sin(x+a)| - C_1$ sin a

5. $\frac{x}{2} + \frac{1}{2}log\,|cos\,x + sin\,x| + C$

6. $\frac{x}{2} + \frac{1}{2}log\,|cos\,x + sin\,x| + C$

7. $2\sqrt{1+sin\,x} + C$

8. $2\sqrt{1+sin\,x} + C$

9. $2\sqrt{tan\,x} + C$

Exercise-7.3

10. $\frac{x}{2} - \frac{1}{8}sin\,(4x+10) + C$

11. $-\frac{1}{2}\cos(2x+1) + \frac{1}{6}cos^3(2x+1) + C$

12. $\frac{1}{2}\left[\frac{1}{4}sin\,4x - \frac{1}{12}sin\,12x\right] + C$

13. $2(sin\,x + x\cos\alpha) + C$

14. $-\frac{1}{cos\,x+sin\,x} + C$

15. $\frac{1}{6}sec^3 2x - \frac{1}{2}sec\,2x + C$

16. $log\,|tan\,x| + \frac{1}{2}tan^2$ x + C

17. $log\,|cos\,x + sin\,x\,| + C$

18. $\frac{1}{sin\,(a-b)}log\,\left|\frac{cos(x-a)}{cos\,(x-b)}\right| + C$

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Exercise-7.4

19. $Ans. \frac{1}{17} log \left|\frac{6x-4}{6x+30}\right| + C_1$

20. $Ans. -\sqrt{5-4x-x^2} + \sin^{-1}\frac{x+2}{3} + C$

21. $Ans. \frac{1}{6}\tan^{-1}\left(\frac{3x+1}{2}\right) + C$

22. $Ans. \frac{1}{2} log\,|x^2-2x-5| + \frac{2}{\sqrt{6}}\log\left|\frac{x+1-\sqrt{6}}{x-1+\sqrt{6}}\right| + C$

Exercise-7.5

23. $Ans. x - 5\ log\,|x-2| + 10\ log\,|x-3| + C$

24. $Ans. 11.4\ log\left|\frac{x+1}{x+3}\right| + \frac{5}{2(x+1)} + C$

25. $Ans. 3\ log\,(2-sin\,\phi) + \frac{4}{2-sin\,\phi} + C$

26. $Ans. \frac{3}{5} log\,|x+2| + \frac{1}{5} log\,|x^2+1| + \frac{1}{5}\tan^{-1} x + C$

27. $Ans. log\,|x-1| - 5\ log\,|x-2| + 4\ log\,|x-3| + C$

28. $Ans. x + \frac{2}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}} - \tan^{-1}\frac{x}{2} + C$

29. $Ans. \frac{1}{2} log\left(\frac{x^2+1}{x^2+3}\right) + C$

30. $Ans. \frac{1}{4} log\left|\frac{x^4-1}{x^4}\right| + C$

31. $Ans. log\left(\frac{e^x-1}{e^x}\right) + C$

Exercise-7.6

32. $Ans. -[\sqrt{1-x^2}\cos^{-1} x + x] + C$

33. $Ans. \frac{e^x}{1+x} + C$

34. $Ans. e^x\ tan\frac{x}{2} + C$

35. $Ans. \frac{e^x}{(x-1)^2} + C$

36. $Ans. \frac{e^{2x}}{5}(2\ sin\,x - cos\,x) + C$

37. $Ans. 2x\ \tan^{-1} x - log\,(1+x^2) + C$

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Exercise-7.7

38. $Ans. \frac{(x+2)}{2}\sqrt{x^2+4x-5} - \frac{9}{2} log\left|x+2+\sqrt{x^2+4x-5}\right| + C$

Exercise-7.10

39. $Ans. \frac{11}{4}$

40. $Ans. \frac{\pi^2}{4}$

41. $Ans. \frac{\pi}{12}$

42. $Ans. \frac{-\pi}{2} log\ 2$

43. $Ans. \frac{\pi}{4}$

44. $Ans. 29$

45. $Ans. \frac{1}{(n+1)(n+2)}$

46. $Ans. \frac{\pi}{2} log \frac{1}{2}$

47. $Ans. -\pi log\ 2$

Miscellaneous Exercise

48. $Ans. \frac{x^2}{2} + x + \frac{1}{2} log|x-1| - \frac{1}{4}\ log\ (x^2+1) - \frac{1}{2}\tan^{-1} x + C$

49. $Ans. x\ log\ (log\ x) - \frac{x}{log\ x} + C$

50. $Ans. \sqrt{2}\tan^{-1}\left(\frac{tan\ x-1}{\sqrt{2}\ tan\ x}\right) + C$

51. $Ans. \frac{\pi^2}{2\ ab}$

52. $Ans. sin\ a\ log\ |sin\ (x-a)| + x\ cos\ a + C$

53. $Ans. \frac{1}{sin\ (a-b)} log\ \left|\frac{cos\ (x+b)}{cos\ (x+a)}\right| + C$

54. $Ans. -2\sqrt{1-x} + \cos^{-1}\sqrt{x} + \sqrt{x-x^2} + C$

55. $Ans. \frac{1}{2}[x\ \cos^{-1} x - \sqrt{1-x^2}] + C$

56. $Ans. -\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{\frac{3}{2}}\left[log\left(1+\frac{1}{x^2}\right) - \frac{2}{3}\right] + C$

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57. $Ans. \frac{1}{40} log\ 9$

58. $Ans.\ \frac{\pi}{2} - 1$

59. $Ans.\ \frac{\pi}{2}(\pi - 2)$

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Chapter-8 Application of Integrals

Exercise-8.1 & Other Extra Questions

1. Find the area of the region bounded by $y^2 = 9x$, $x = 2$, $x = 4$ and the x-axis in the first quadrant. **(Old NCERT)**
2. Find the area of the region in the first quadrant enclosed by x-axis, line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$. **(Old NCERT)**
3. The area between $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, find the value of a. **(Old NCERT)**
4. Find the area of the region bounded by the parabola $y = x^2$ and $y = |x|$.**(Old NCERT)**
5. Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$. **(Old NCERT)**

Related to Exercise-8.2 (Old NCERT)

6. Using integration finds the area of region bounded by the triangle whose vertices are (1, 0), (2, 2) and (3, 1). **(Old NCERT)**
7. Using integration find the area of the triangular region whose sides have the equations $y = 2x + 1$, $y = 3x + 1$ and $x = 4$. **(Old NCERT)**

Related to Miscellaneous Exercise & Other Questions

8. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$ **(Old NCERT)**
9. Using the method of integration find the area bounded by the curve $|x| + |y| = 1$. **(Old NCERT)**
10. Find the area bounded by curves $\{(x, y) : y \geq x^2$ and $y = |x|\}$**(Old NCERT)**

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Answers

Exercise-8.1

1. $Ans. 16 - 4\sqrt{2}$
2. $Ans. \frac{\pi}{3}$
3. $Ans. (4)^{\frac{2}{3}}$
4. $Ans. \frac{1}{3}$
5. $Ans. \frac{9}{8}$

Exercise-8.2

6. **7/2**
7. **8 Square Units**

Miscellaneous Exercise

8. $Ans. \frac{3}{2}(\pi - 2)$
9. $Ans. 2$
10. $Ans. \frac{1}{3}$

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Chapter-9 DIFFERENTIAL EQUAITONS

Exercise-9.1

1. Determine order and degree $\frac{d^4y}{dx^4} + sin$ (y" `) = 0 **[Q 1]**

2. Determine order and degree $(y''')^2 + (y'')^3 + (y')^4 + y^5 = 0$ **[Q 6]**

Exercise-9.2

3. Verify that the functions $y = e^{-3x}$ is a solution of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$ **[Example 2]**

Exercise-9.3

4. Find the particular solution of the differential equation $\frac{dy}{dx} = -4xy^2$ given that y = 1, when x = 0. **[Example 6]**

5. In a bank, principal increases continuously at the rate of 5% per year. In how many years Rs 1000 double itself? **[Example 9]**

6. Find the particular solution satisfying the given condition:

$(x^3 + x^2 + x + 1)\frac{dy}{dx} = 2x^2 + x$; y = 1 when x = 0 **[Q 11]**

7. Find the particular solution satisfying the given condition:

$\frac{dy}{dx} =$ y tan x; y = 1 when x = 0 **[Q 14]**

8. For the differential equation xy $\frac{dy}{dx} = (x + 2)(y + 2)$, find the solution curve passing through the point (1, -1) **[Q 16]**

9. Find the equations of a curve passing through the point (0, -2) given that at any point (x, y) on the curve, the product of the slope of its tangent and y coordinate of the point is equal to the x coordinate of the point. **[Q 17]**

Exercise-9.4

10. Show that the differential equation x $\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$ is homogeneous and solve it. **[Example 11]**

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11. Show that the differential equation 2y $e^{\frac{x}{y}}$ dx + $\left(y - 2x\, e^{\frac{x}{y}}\right) dy = 0$ *is* homogeneous and find its particular solution, given that x = 0 when y = 1. **[Example 12]**

12. Show that the family of curves for which the slope of the tangent at any point (x, y) on it is $\frac{x^2+y^2}{2xy}$, is given by $x^2 - y^2 = cx$ **[Example 13]**

13. Show that the given differential equation is homogeneous and solve each of them.

x dy – y dx = $\sqrt{x^2 + y^2}\, dx$ **[Q 6]**

14. Show that the given differential equation is homogeneous and solve each of them.

$\left\{x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right)\right\} y\, dx = \left\{y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right)\right\} x\, dx$ **[Q 7]**

15. Find the particular solution satisfying the given condition:

$x^2dy + (xy + y^2)\, dx = 0$; y = 1 where x = 1. **[Q 12]**

16. Find the particular solution satisfying the given condition:

$2xy + y^2 - 2x^2\frac{dy}{dx} = 0$; y = 2 where x = 1 **[Q 15]**

Exercise-9.5

17. find the particular solution of the differential equation :

$\frac{dy}{dx} + y \cot x = 2x + x^2 \cot x\ (x \neq 0) given\ that\ y = 0\ where\ x = \frac{\pi}{2}$. **[Example 17]**

18. Find the equation of a curve passing through the point (0, 1). If the slope of the tangent to the curve at any point (x, y) is equal to the sum of the x coordinate (abscissa) and the product of the x coordinate and y coordinate (ordinate) of that point. **[Example 18]**

19. For each of the differential equations, find the general solution:

x log x $\frac{dy}{dx} + y = \frac{2}{x} \log x$ **[Q 7]**

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20. For each of the differential equations, find the general solution:

$(1 + x^2)\, dy + 2xy\, dx = \cot x\, dx$ ($x \neq 0$) **[Q 8]**

21. For each of the differential equations, find the particular solution satisfying the given condition:

$\frac{dy}{dx} + 2y \tan x = \sin x\, ; y = 0\ when\ x = \frac{\pi}{3}$ **[Q 13]**

22. For each of the differential equations, find the particular solution satisfying the given condition:

$(1 + x^2)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$; y = 0 here x = 1 **[Q 14]**

23. For each of the differential equations, find the particular solution satisfying the given condition:

$\frac{dy}{dx} - 3y \cot x = \sin 2x; y = 2\ where\ x = \frac{\pi}{2}$ **[Q 15]**

24. Find the equations of a curve passing through the origin given that the slope of the tangent to the curve at any point (x, y) is equal to the sum of the coordinates of the point. **[Q 16]**

25. Find the equation of a curve passing through the point (0, 2) given that the sum of the coordinates of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at that point by 5. **[Q 17]**

Miscellaneous Exercise

26. Solve the differential equation $(x\, dy - y\, dx)y \sin\left(\frac{y}{x}\right) = (\, y\, dx + x\, dy)x \cos\left(\frac{y}{x}\right)$.**[Example 21]**

27. Solve the differential equation $(\tan^{-1} y - x)dy = (1 + y^2)dx$. **[Example 22]**

28. Find the equation of the curve passing through the point $\left(0, \frac{\pi}{4}\right)$whose differential equation is sin x cos y dx + cos x sin y dy = 0. **[Q 6]**

29. Find the particular solution of the differential equation $(1 + e^{2x})$ dy + $(1 + y^2)\, e^x$ dx = 0, given that y = 1 when x = 0. **[Q 7]**

30. Solve the differential equation $y\, e^{x/y}$ dx = $\left(xe^{\frac{x}{y}} + y^2\right) dy$ $(y \neq 0)$. **[Q 8]**

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31. Find a particular solution of the differential equation $\frac{dy}{dx} + y \cot x = 4\, x\, cosec\, x\ (x \neq 0)$. Given that y = 0 when x = $\frac{\pi}{2}$. **[Q 11]**

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Answers

Exercise-9.1

1. order 4 : degree not defined

2. order 3: degree 2

Exercise-9.3

4. $y = \frac{1}{2x^2+1}$

5. $t = 20\ log_e 2$

6. $y = \frac{1}{4} log\ [(x+1)^2(x^2+1)^3] - \frac{1}{2}\tan^{-1} x + 1$

7. $Ans. y = sec\ x$

8. $Ans.$ $y - x + 2 = \log (x^2 (y + 2)^2)$

9. Ans. $y^2 - x^2 = 4$.

Exercise-9.4

10. $Ans.$ $\sin\left(\frac{y}{x}\right) = log|Cx|$

11. $Ans.$ $2e^{x/y} + \log |y| = 2$

12. $Ans.$ $(y^2 - x^2) = \pm C_1 x$ or $x^2 - y^2 = Cx$

13. $Ans.$ $y+\sqrt{x^2+y^2} = Cx^2$

14. $Ans. xy\ cos\ \left|\frac{y}{x}\right| = C$

15. $Ans. y + 2x = 3x^2y$

16. $Ans. y = \frac{2x}{1-log\ |x|} (x \neq 0, x \neq e)$

Exercise-9.5

17. $Ans. y = x^2 - \frac{\pi^2}{4sin\ x}$

18. $Ans. y = -1 + 2e^{\frac{x^2}{2}}$

19. $Ans. y\ log\ x = \frac{-2}{x}(1 + log\ x) + C$

20. $Ans. y = (1+x^2)^{-2} log\ |sin\ x| + C\ (1 + x^2)^{-1}$

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21. $Ans. y = cos\ x - 2\ cos^2$ x

22. $Ans. y\ (1 + x^2)$ = $\tan^{-1}$ x - $\frac{\pi}{4}$

23. $Ans. y = 4\ sin^3$ x – 2 $\sin^2$ x

24. $Ans. x + y + 1 = e^x$

25. $Ans. y = 4 - x - 2\ e^x$

Miscellaneous Exercise

26. $Ans. sec\ \left(\frac{y}{x}\right) =\ C\ xy$

27. $Ans. x = (tan^{-1}$ y – 1) + C $e^{-\tan^{-1} y}$

28. $Ans. cos\ y = \frac{sec\ x}{\sqrt{2}}$

29. $Ans. \tan^{-1} y + \tan^{-1}(e^x) = \frac{\pi}{2}$

30. $Ans. e^{\frac{x}{y}} = y + C$

31. $Ans. y\ sin\ x = 2x^2 - \frac{\pi^2}{2} (sin\ x\ \neq 0)$

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Chapter-10 Vector Algebra

Exercise-10.2

1. Find the unit vector in the direction of the sum of the vectors, $\vec{a} = 2\hat{\imath} + 2\hat{\jmath} - 5\hat{k}$ and $\vec{b} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$.**[Example 8]**
2. Write the direction ratios of the vector $\vec{a} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$ and hence calculate its direction cosines. **[Example 9]**
3. Show that the point A$(2\hat{\imath} - \hat{\jmath} + \hat{k}), B\left(\hat{\imath} - 3\hat{\jmath} - 5\hat{k}\right), C(3\hat{\imath} - 4\hat{\jmath} - 4\hat{k})$ are the vertices of a right angled triangle. **[Example 12]**
4. Find a vector in the direction of vector $5\hat{\imath} - \hat{\jmath} + 2\hat{k}$ which has magnitude 8 units. **[Q 10]**
5. Find the direction cosines of the vector joining the point A(1, 2, -3) and B (-1, -2, 1), directed from A to B. **[Q 13]**
6. Show that the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$ is equally inclined to the axes OX, OY and OZ. **[Q 14]**
7. Find the position vector of the mid point of the vector joining the points P(2, 3, 4) and Q(4, 1, -2). **[Q 16]**
8. Show that the points A, B and C with position vectors, $\vec{a} = 3\hat{\imath} - 4\hat{\jmath} - 4\hat{k}$, $\vec{b} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\vec{c} = \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$, respectively from the vertices of a right angled triangle. **[Q 17]**

Exercise-10.3

9. Find angle 'θ' between the vectors $\vec{a} = \hat{\imath} + \hat{\jmath} - \hat{k}$ and $\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$. **[Example 14]**
10. If $\vec{a} = 5\hat{\imath} - \hat{\jmath} - 3\hat{k}$ and $\vec{b} = \hat{\imath} + 3\hat{\jmath} - 5\hat{k}$, then show that the vectors $\vec{a} + \vec{b} and\ \vec{a} - \vec{b}$ are perpendicular. **[Example 15]**
11. Find $|\vec{a} - \vec{b}|$, if two vectors $\vec{a}\ and\ \vec{b}$ are such that $|\vec{a}| = 2, \left|\vec{b}\right| = 3$ and $\vec{a}.\vec{b} = 4$. **[Example 17]**
12. If $\vec{a}$ is a unit vector and $(\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 8$, then find $|\vec{x}|$. **[Example 18]**
13. Find the projection of the vector $\hat{\imath} + 3\hat{\jmath} + 7\hat{k}$ on the vector $7\hat{\imath} - \hat{\jmath} + 8\hat{k}$. **[Q 4]**

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14. Find the magnitude of tow vectors $\vec{a}\ and\ \vec{b}$, having the same magnitude and such that the angle between them is 60^0 and their scalar product is ½ . **[Q 8]**

15. If $\vec{a} = 2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}, \vec{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{c} = 3\hat{\imath} + \hat{\jmath}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of λ. **[Q 10]**

16. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$. **[Q 13]**

17. If the vertices A, B, C of a triangle ABC are (1, 2, 3), (-1, 0, 0), (0, 1, 2), respectively, then find $\angle ABC$. $|\angle ABC$ is the angle between the vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}|$.**[Q 15]**

Exercise-10.4

18. Find the area of a triangle having the points A(1, 1, 1), B(1, 2, 3) and C(2, 3,1) as its vertices. **[Example 24]**

19. Find the area of a parallelogram whose adjacent sides are given by the vectors $\vec{a} = 3\hat{\imath} + \hat{\jmath} + 4\hat{k}$ and $\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$. **[Example 25]**

20. Find a unit vector perpendicular to each of the vector $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$, where $\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$. **[Q 2]**

21. If a unit vector $\vec{a}$ makes angles $\frac{\pi}{3}$ with $\hat{\imath}$, $\frac{\pi}{4}$ with $\hat{\jmath}$ and an acute angle θ with $\hat{k}$, then find θ and hence, the components of $\vec{a}$. **[Q 3]**

Miscellaneous Exercise

22. Let $\vec{a}, \vec{b}\ and\ \vec{c}$ be three vectors such that $|\vec{a}| = 3, \left|\vec{b}\right| = 4, |\vec{c}| = 5$ and each of the them being perpendicular to the sum of the other two, find $|\vec{a} + \vec{b} + \vec{c}|$. **[Example 28]**

23. Three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Evaluate the quantity $\mu = \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$, if $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{c}| = 2$. **[Example 29]**

24. If with reference to the right handed system of mutually perpendicular unit vectors $\hat{\imath}, \hat{\jmath} and\ \hat{k}, \vec{\alpha} = 3\hat{\imath} - \hat{\jmath}, \vec{\beta} = 2\hat{\imath} + \hat{\jmath} - 3\hat{k}$, then express $\vec{\beta}$ in the form $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$. **[Example 30]**

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25. A girl walks 4 km towards west, then the walks 3 km is a direction 30^0 east of north and stops. Determine the girl's displacement form her initial point of departure. **[Q 3]**

26. Find a vector of magnitude 5 units, and parallel to the resultant of the vectors $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$ and $\vec{b} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$. **[Q 6]**

27. The two adjacent sides of a parallelogram are $2\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ and $\hat{\imath} - 2\hat{\jmath} - 3\hat{k}$. Find the unit vector parallel to its diagonal.Also,find its area.**[Q 10]**

28. Let $\vec{a} = \hat{\imath} + 4\hat{\jmath} + 2\hat{k}, \vec{b} = 3\hat{\imath} - 2\hat{\jmath} + 7\hat{k}$ and $\vec{c} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c}.\vec{d} = 15$. **[Q 12]**

29. The scalar product of the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ and $\lambda\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ is equal to one. Find the value of λ. **[Q 13]**

30. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}, \vec{b}\ and\ \vec{c}$. **[Old NCERT]**

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Answers

Exercise 10.2

1. $\frac{4}{\sqrt{29}}\hat{\imath} + \frac{3}{\sqrt{29}}\hat{\jmath} - \frac{2}{\sqrt{29}}\hat{k}$

2. $\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}$

3.

4. $\frac{40}{\sqrt{30}}\hat{\imath} - \frac{8}{\sqrt{30}}\hat{\jmath} + \frac{16}{\sqrt{30}}\hat{k}$

5. $-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$

6.

7. $3\hat{\imath} + 2\hat{\jmath} + \hat{k}$

8.

Exercise 10.3

9. $\cos^{-1}\left(-\frac{1}{3}\right)$

10.

11. $\sqrt{5}$

12. 3

13. $\frac{60}{\sqrt{114}}$

14. $|\vec{a}| = 1,\ |\vec{b}| = 1$

15. 8

16. $\frac{-3}{2}$

17. $\cos^{-1}\left(\frac{10}{\sqrt{102}}\right)$

Exercise 10.4

18. $\frac{1}{2}\sqrt{21}$

19. $\sqrt{42}$

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20. $\pm\frac{2}{3}\hat{\imath}\mp\frac{2}{3}\hat{\jmath}\mp\frac{1}{3}\hat{k}$

21. $\frac{\pi}{3};\frac{1}{2},\frac{1}{\sqrt{2}},\frac{1}{2}$

Miscellaneous Exercise

22. $5\sqrt{2}$

23.

24. $\vec{\beta}_1\frac{3}{2}\hat{\imath}-\frac{1}{2}\hat{\jmath}$ and $\vec{\beta}_2\frac{1}{2}\hat{\imath}+\frac{3}{2}\hat{\jmath}-3\hat{k}$

25. $-\frac{5}{2}\hat{\imath}+\frac{3\sqrt{3}}{2}\hat{\jmath}$

26. $\frac{3}{2}\sqrt{10}\hat{\imath}+\frac{\sqrt{10}}{2}\hat{\jmath}$

27. $\frac{1}{7}\left(3\hat{\imath}-6\hat{\jmath}+2\hat{k}\right);11\sqrt{5}$

28. $\frac{1}{3}\left(160\hat{\imath}-5\hat{\jmath}+70\hat{k}\right)$

29. 1

30.

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Chapter-11 THREE DIMENSIONAL GEOMETRY

Exercise-11.1

1. If a line makes angles 90^0, 135^0, 45^0 with x, y and z-axis respectively, find its direction cosines. **[Q 1]**

2. Find the direction cosines of a line which makes equal angles with the coordinates axes. **[Q 2]**

3. Find the direction cosines of the sides of the triangle whose vertices are (3, 5, -4), (-1, 2, 2) and -5, -5, -2) **[Q 4]**

Exercise-11.2

4. Find the angle between the pair of line given by

$\vec{r} = \hat{3\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$ and **[Example 7]**

$\vec{r} = 5\hat{\imath} - 2\hat{\jmath} + \mu(3\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$

5. Find the angle between the pair of lines

$\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$ and **[Example 8]**

$\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$

6. Find the shortest distance between the lines l_1 and l_2 whose vector equations are

$\vec{r} = \hat{\imath} + \hat{\jmath} + \lambda(\hat{2\imath} - \hat{\jmath} + \hat{k})$ and

$\vec{r} = 2\hat{\imath} + \hat{\jmath} - \hat{k} + \mu(3\hat{\imath} - 5\hat{\jmath} + 2\hat{k})$ **[Example 9]**

7. Find the distance between the lines l_1 and l_2 given by

$\vec{r} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$ and **[Example 10]**

$\vec{r} = 3\hat{\imath} + 3\hat{\jmath} - 5\hat{k} + \mu(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$

8. Find the equation of the line which passes through the point (1, 2, 3) and is parallel to the vector $3\hat{\imath} + 2\hat{\jmath} - 2\hat{k}$. **[Q 4]**

9. Find the equation of the line in vector and in Cartesian form that passes through the point with position vector $2\hat{\imath} - \hat{\jmath} + 4\hat{k}$ and is in the direction $\hat{\imath} + 2\hat{\jmath} - \hat{k}$. **[Q 5]**

10. Find the Cartesian equation of the line which passes through the point (-2, 4, -5) and parallel to the line given by $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$. **[Q 6]**

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11. Find the vector and the Cartesian equations of the line that passes through the points (3, -2, -5), (3, -2, 6). **[Old NCERT]**

12. Find the values of p so that liens $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles. **[Q 10]**

13. Find the shortest direction distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$. **[Q 13]**

Miscellaneous Exercise

14. If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{x-6}{-5}$ are perpendicular, find the value of k. **[Q 3]**

15. Find the shortest distance between lines $\vec{r} = 6\hat{\imath} + 2\hat{\jmath} + 2\hat{k} + \lambda(\hat{\imath} - 2\hat{\jmath} + 2\hat{k})$ and

$\vec{r} = -4\hat{\imath} - \hat{k} + \mu(3\hat{\imath} - 2\hat{\jmath} - 2\hat{k})$ **[Q 4]**

16. Find the vector equation of the line passing through the point (1, 2, -4) and perpendicular to the two lines: $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$ **[Q 5]**

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Answers

Exercise-11.1

1. $0, \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

2. $\pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}$

3. $\frac{-2}{\sqrt{17}}, \frac{-2}{\sqrt{17}}, \frac{3}{17}; \frac{-2}{\sqrt{17}}, \frac{-3}{\sqrt{17}}, \frac{-2}{\sqrt{17}}; \frac{4}{\sqrt{42}}, \frac{5}{\sqrt{42}}, \frac{-1}{\sqrt{42}}$

Exercise-11.2

4. $\cos^{-1}\left(\frac{19}{21}\right)$

5. $\cos^{-1}\left(\frac{8\sqrt{3}}{15}\right)$

6. $\frac{10}{\sqrt{59}}$

7. $\frac{\sqrt{293}}{7}$

8. $\vec{r} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} + \lambda(3\hat{\imath} + 2\hat{\jmath} - 2\hat{k}), where\ \lambda\ is\ an\ real\ number$

9. $\vec{r} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k} + \lambda(\hat{\imath} + 2\hat{\jmath} - \hat{k}), and\ caresian\ form\ is\ \frac{x-2}{1} = \frac{y+1}{2} = \frac{z-4}{-1}$

10. $\frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}$

11. vector equation of the line : $\vec{r} = 3\hat{\imath} - 2\hat{\jmath} - 5\hat{k} + \lambda(11\hat{k})$ Cartesian equation of the line: $\frac{x-3}{0} = \frac{y+2}{0} = \frac{z+5}{11}$

12. 70/11

13. $2\sqrt{29}$

Miscellaneous Exercise

14. $k = \frac{-10}{7}$

15. 9

16. $\vec{r} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$

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Chapter 12 Linear Programming

Exercise 12.1

1. Maximize Z = 5x + 3y
Subject to $3x + 5y \leq 15, 5x + 2y \leq 10, x \geq 0, y \geq 0$. **[Q 3]**

2. Minimize Z = 3x + 5y
Such that $x + 3y \geq 3, x + y \geq 2, x, y \geq 0$. **[Q 4]**

3. Minimize and maximize Z = 5x + 10y
Subject to $x + 2y \leq 120, x + y \geq 60, x - 2y \geq 0, x, y \geq 0$. **[Q 7]**

4. Minimize and maximize Z = x + 2y
Subject to $x + 2y \geq 100, 2x - y \leq 0, 2x + y \leq 200; x, y \geq 0$. **[Q 8]**

5. Maximize Z = -x + 2y, subject to the constraints: **[Q 9]**
$x \geq 3, x + y \geq 5, x + 2y \geq 6, y \geq 0$.

6. Maximize Z = x + y, subject to $x - y \leq -1, -x + y \leq 0, x, y \geq 0$. **[Q 10]**

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Chapter-13 PROBABILITY

Exercise-13.1

1. A family has two children. What is the probability that both the children are boys given that at least one of them is a boy? **[Example 2]**
2. A die is thrown twice and the sum of the numbers appearing is observed to be 6. What is the conditional probability that the number 4 has appeared at least once? **[Example 6]**
3. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls give that (i) the youngest is a girl, (ii) at least one is a girl? **[Q 12]**
4. An instructor has a question bank consisting of 300 easy True/False questions, 200 difficult True/False questions, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a questions is selected at random form the question bank, what is the probability that it will be an easy question given that it is a multiple choice questions? **[Q 13]**
5. Given that the two numbers appearing on throwing two dice are different. Find the probability of the event 'the sum of numbers on the dice is 4' **[Q 14]**
6. Consider the experiment of throwing a die, if a multiple of 3 comes up, throw the die again and if any other number comes, toss a coin. Find the conditional probability of the event 'the coin shows a tail', given that 'at least one die shows a 3'. **[Q 15]**

Exercise-13.2

7. An unbiased die is thrown twice. Let the event A be 'odd number on the first throw and B the event ' odd number on the second throw'. Check the independence of the events A and B. **[Example 11]**
8. Let E and F be events with P(E) = 3/5 , P(F) = 3/10 and P(E ∩ F) = 1/5. Are E and F independent? **[Q 6]**

9. Given two independent events A and B such that P(a) = 0.3, P(B) = 0.6 find

(a) P(A and B)

(b) P(A and not B)

(c) P(A or B)

(d) P(neither A nor B) **[Q 11]**

10. A die is tossed thrice. Find the probability of getting an odd number at least once. **[Q 12]**

11. Probability of solving specific problem independently by A and B are ½ and 1/3 respectively. If both try to solve the problem independently, find the probability that

(a) The problem is solved

(b) Exactly one of them solves the problem **[Q 14]**

12. In a hostel, 60% of the students read Hindi news paper, 40% read English news paper and 20% read both Hindi and English news papers. A student is selected at random

(a) Find the probability that she reads neither Hindi nor English news papers.

(b) If she reads Hindi news paper, find the probability that she read English news paper.

(c) If she reads English news paper, find the probability that she reads Hindi news paper. **[Q 16]**

Exercise-13.3

13. A person has undertaken a construction job. The probabilities are 0.65 that there will be strike, 0.80 that the construction job will be completed on time if there is no strike, and 0.32 that the construction job will be completed on time if there is a strike. Determine the probability that the construction job will be completed on time. **[Example 15]**

14. Suppose that the reliability of HIV test is specified as follows:

Of people having HIV, 90% of the test detect the disease but 10% go undetected. Of people free of HIV ,99% of the test are judged HIV-ive but 1% are diagnosed as showing HIV+ive. Form a large population of which only

0.1% have HIV, one person is selected at random, given the HIV test, and the pathologist reports him/her as HIV +ve. What is the probability that the person actually has HIV? **[Example 18]**

15. A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively 3/10, 1/5, 1/10 and 2/5. The probabilities that he will be late are ¼, 1/3 and 1/12, if he comes by train, bus and scooter respectably, but he comes by other means of transport, then he will not be late. When he arrives, he is late, what is the probability that the comes by train? **[Example 20]**
16. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six. **[Example 21]**
17. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let ¾ be the probability that he knows the answer and ¼ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability ¼. What is the probability that the student known the answer given that he answered it correctly? **[Q 4]**
18. A laboratory blood test is 99% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false positive result for 0.5% of the healthy person tested (i.e. if a healthy person is tested, then with probability 0.005, the test will imply he has the disease). It 0.1 percent of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive? **[Q 5]**
19. There are three coins. One is a two headed coin (having head on both faces), another is the biased coin that comes up heads 75% of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads, what is the probability that it was the two headed coin? **[Q 6]**
20. A factory has tow machines A and B. past record shows that machine A produced 60% of the items of output and machine B produced 40% of the items. Further, 2% of the items produced by machine A and 1% produced by machine B were defective. All the items are put into one stockpile and then one

item is chosen at random from this and is found to be defective. What is the probability that is was produced by machine B? **[Q 8]**

21. Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads, if she gets 1,2,3 or 4 she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1,2,3, or 4 with the die? **[Q 10]**

22. A card from a pack of 52 cards is lost. From the remaining cards of the pack two cards are drawn and are found to be both diamonds. Find the probability of the lost card being diamond. **[Q 12]**

Related to Exercise-13.4(Old NCERT)

23. Find the probability distribution of number of doublets in three throws of a pair of dice. **[Example 25] (Old NCERT)**

24. Let a pair of dice be thrown and the random variable X be the sum of the numbers that appear on the two dice. Find the mean or expectation of X. **[Example 27] (Old NCERT)**

25. Two cards are drawn simultaneously (or successively without replacement) form a well shuffled pack of 52 cards. Find the mean, variance and standard deviation of the number of kings. **[Example 29] (Old NCERT)**

26. Find the probability distribution of the number of successes in two tosses of a die where a success is defined as

(a) Number greater than 4

(b) Six appears on at least one die. **[Q 5] (Old NCERT)**

27. From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs. **[Q 6] (Old NCERT)**

28. A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails. **[Q 7] (Old NCERT)**

29. A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	K^2	$2k^2$	$7k^2+k$

Determine

(a) k

(b) P(X < 3)

(c) P(X> 6)

(d) P(0< X<3) **[Q 8] (Old NCERT)**

30. Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X. **[Q 11] (Old NCERT)**

Miscellaneous Exercise

31. A and B throw a die alternatively till one of them gets a '6' and wins the game. Find their respective probabilities of winning, if A starts first. **[Example 23]**

32. In a game, a man wins a rupee for a six and losses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins/loses. **[Old NCERT]**

33. Assume that the chances of a patient having a heart attack is 40%. It is also assumed that a mediation and yoga course reduce the risk of heart attack by 30% and prescription of certain drug reduces its chance by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga? **[Q 7]**

34. If each element of a second order determinant is either zero or one, what is the probability that the value of determinant is positive? (Assume that the individual entries of the determinant are chosen independently. Each value being assumed with probability ½) **[Q 8]**

35. Bag I contains 3 red and 4 black balls and bag II contains 4 red and 5 black balls. One ball is transferred from bag I to Bag II and then a ball is drawn from

Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black. **[Q 10]**

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Answers

Exercise-13.1

1. 1/3

2. 2/5

3. (i) ½ (ii) 1/3

4. 5/9

5. 1/15

6. 0

Exercise-13.2

7. A and B are independent events

8. E and F are not independent

9.

(a) 0.18

(b) 0.12

(c) 0.72

(d) 0.28

10. 7/8

11. (a) 2/3 (b) ½

12. (a) 1/5 (b) 1/3 (c) ½

Exercise-13.3

13. 0.488

14. 0.083

15. ½

16. 3/8

17. 12/13

18. 198/1197

19. 4/9

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20. ¼

21. 8/11

22. 11/50

Exercise-13.4

23.

x	0	1	2	3
P(X)	125/216	75/216	15/216	1/216

24. 7

25. 34/221 , $6800/(221)^2$, 0.37

26.

(a)

X	0	1	2
P(X)	4/9	4/9	1/9

(b)

X	0	1
P(X)	25/36	11/36

27.

X	0	1	2	3	4
P(X)	256/625	256/625	96/625	16/625	1/625

28.

X	0	1	2
P(X)	9/16	6/16	1/16

29.

(a) k = 1/10

(b) P(X<3)=3/10

(c) P(X>6)=17/100

(d) P(0<X<3(=3/10

30. 1/3

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Miscellaneous Exercise

31. $\frac{6}{11}, \frac{5}{11}$

32. $\frac{11}{216}$

33. $\frac{14}{29}$

34. $\frac{3}{16}$

35. $\frac{16}{31}$

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